Probability:

pick a random number between 1 and 10

1	2	3	4	5	6	7	8	9	10
|||	||	||	|		~~||||~~	~~||||~~	|	|||	

Defn: Set

A "set" is a collection of things/objects

Ex.

$$S = \{1, 2, 3\}$$

$$\mathbb{N} = \{1, 2, 3, 4, 5, 6, 7, \dots\} = \text{natural numbers}$$

$$\mathbb{Q} = \left\{ \frac{m}{n} : m, n \in \mathbb{N} \right\} = \text{rational numbers}$$

Defn: Set Membership

We say "x is an element/member of S"

denoted $x \in S$ if x is in S.

Ex. $5 \in \mathbb{N}$

$\{1, 2, 3, 4, 5, 6, \dots\}$

$2/3 \in \mathbb{Q}$

Ex. $2/3 \notin \mathbb{N}$ b/c $2/3$ isn't in $\mathbb{N}$

"not in"

<u>Defn: Containment</u>

A is contained in B

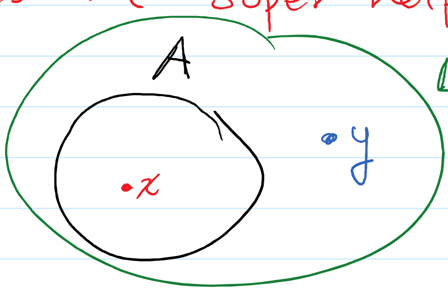
We say a set A is a <u>subset</u> of a set B

denoted $\underline{A \subset B}$

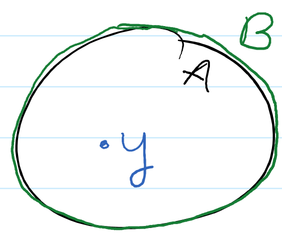
if $x \in A \Rightarrow x \in B$.

"implies"

Pictures are super helpful for prob.



proper subset



improper subset

<u>Ex.</u> $\{1,2,3\} \subset \mathbb{N}$

$\mathbb{N} \not\subset \{1,2,3\}$

not a subset

<u>Ex.</u> $\mathbb{Q} \subset \mathbb{R}$

real numbers

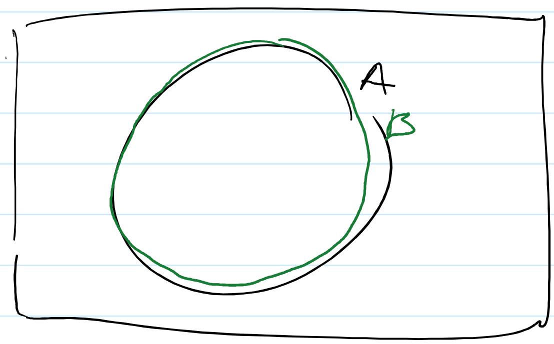
$\mathbb{R} \subset \mathbb{C}$

complex numbers

Defn: Set Equality

We say "A is equal to B" if

$$B \subset A \text{ and } A \subset B.$$



Ex. $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$

$\mathbb{N} \neq \mathbb{R}$ why? $2.2 \in \mathbb{R}$ but $2.2 \notin \mathbb{N}$.

not equal

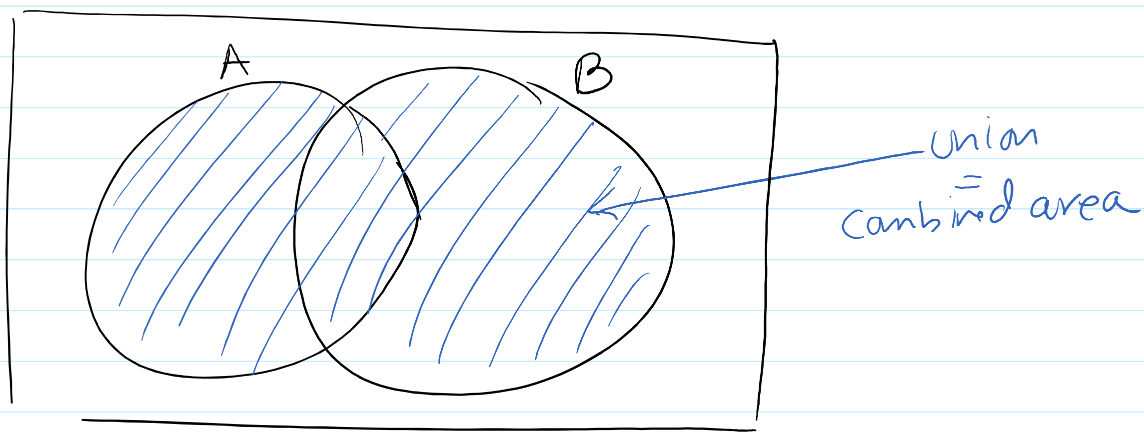
Set Operations

Defn: Union

The union of A and B

"such that"

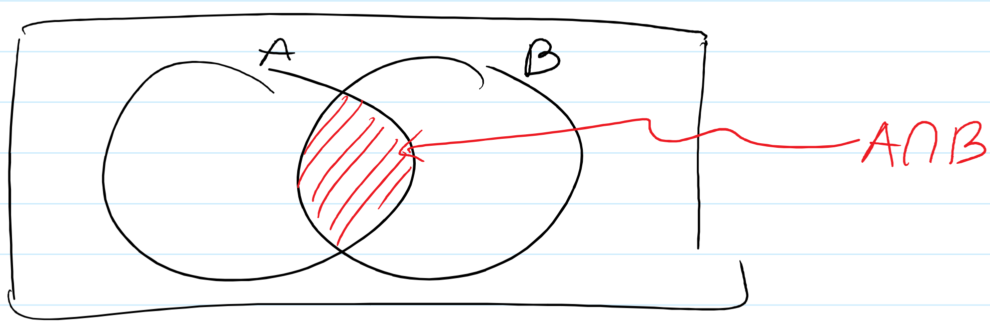
denoted $$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$



Defn: Intersection

The intersection of A and B, denoted

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

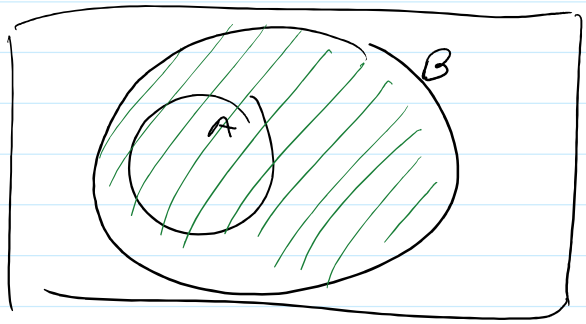


Ex. $A = \{-1, -2, -3, -4, \ldots\}$
$B = \{1, 2, 3, 4, \ldots\}$

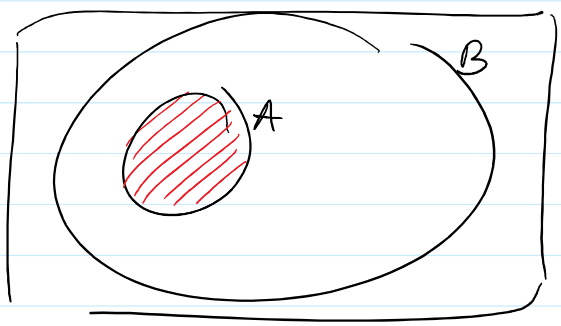
$A \cup B = \{\pm 1, \pm 2, \pm 3, \ldots\}$
$A \cap B = \emptyset$ empty set — the set containing no elements

Theorems:

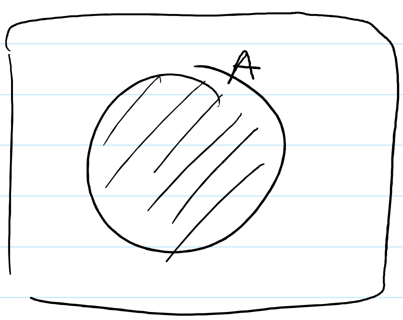
① If $A \subset B$ then $A \cup B = B$



② If $A \subset B$ then $A \cap B = A$



③ $A \cup A = A$; $A \cap A = A$ (Idempotentcy)

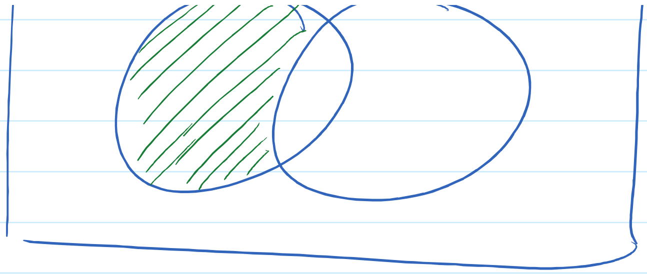


Defn: Set Difference

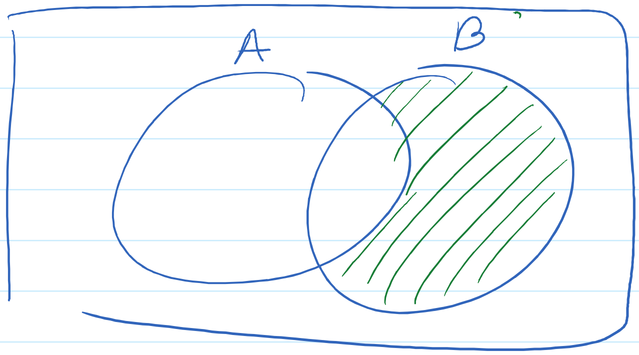
The difference between A and B, denoted

$$A \setminus B = \{x \mid x \in A \text{ but } x \notin B\}$$

$A \setminus B$



$A \setminus B$



$B \setminus A$

Notice that $A \setminus B \neq B \setminus A$ (generally).

Ex. $A = \{1, 2, 3\}$, $B = \{3, 4, 5\}$

$A \setminus B = \{1, 2\}$

$B \setminus A = \{4, 5\}$.

Defn: Set Complements

Assume all sets live in some larger "universe" S

i.e. $A \subset S$

the complement of A, denoted A^c, is defined as

$$A^c = \{x \mid x \notin A\} = \{x \in S \mid x \notin A\}$$

Ex. $A = \{5, 6\}$; $S = \mathbb{N}$

$A^c = \{1, 2, 3, 4, 7, 8, \ldots\}$

$\rightarrow$ $A = \{5, 6\}$; $S = \mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$

all integers

$A^c = \{0, \pm 1, \pm 2, \pm 3, \pm 4, -5, -6, \pm 7, \ldots\}$

Fact: $A^c = S \setminus A$.

Theorems about Set Operations

"AB" = "$A \cap B$"

(1) Commutivity: $A \cup B = B \cup A$

$AB = BA$

(2) Associativity: $(A \cup B) \cup C = A \cup (B \cup C)$

$(AB)C = A(BC)$

(3) Distributivity: $A(B \cup C) = AB \cup AC$

$A \cup (BC) = (A \cup B)(A \cup C)$

(4) De Morgan's Laws:

(i) $(A \cup B)^c = A^c \cap B^c = A^c B^c$

(ii) $(AB)^c = A^c \cup B^c$.

Infinite Set Operations

Countably

Let $A_1, A_2, A_3, A_4, \ldots$ be a seq. of sets

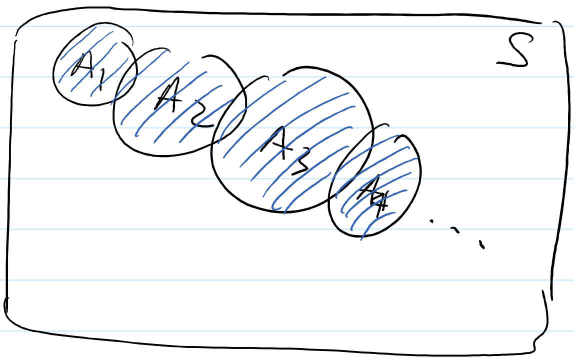
Let $A_1, A_2, A_3, A_4, \ldots$ be a seq. of sets

$A_i \subset S$ for all $i = 1, 2, \ldots$

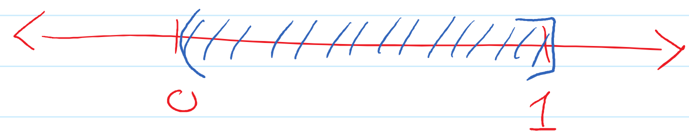
Defn: Union

The union of $\{A_i\}_{i=1}^{\infty}$ is defined as

$$\bigcup_{i=1}^{\infty} A_i \stackrel{\text{def}}{=} \{x \in S \mid x \in A_i \text{ for some } i\}.$$

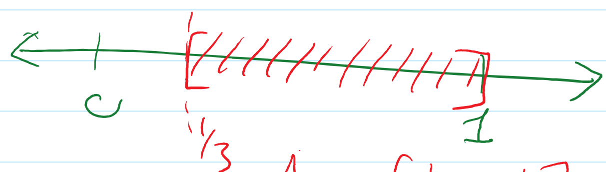


Ex. $S = (0, 1]$



$A_i = [1/i, 1]$

$i = 1, 2, \ldots$



$A_3 = [1/3, 1]$

$$\boxed{\bigcup_{i=1}^{\infty} A_i = S = (0,1]}$$

$A_1 = \{1\}$

$A_2 = [1/2, 1]$

$A_3 = [1/3, 1]$

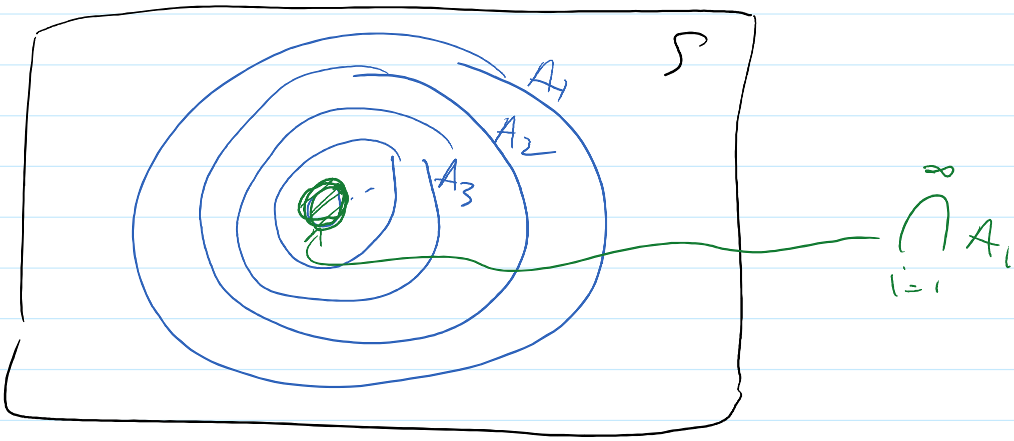
$\vdots$

Defn: Infinite Intersection

The intersection of $\{A_i\}_{i=1}^{\infty}$ is

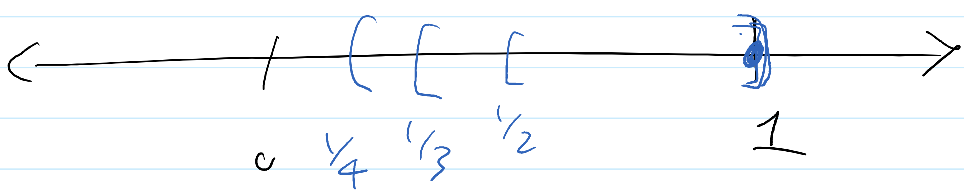
The intersection of $\{A_i\}_{i=1}^{\infty}$ is

$$\bigcap_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \text{ \underline{for all} } i\}$$



Ex. $A_i = [\frac{1}{i}, 1]$ as prev. then

$$\bigcap_{i=1}^{\infty} A_i = \{1\}$$

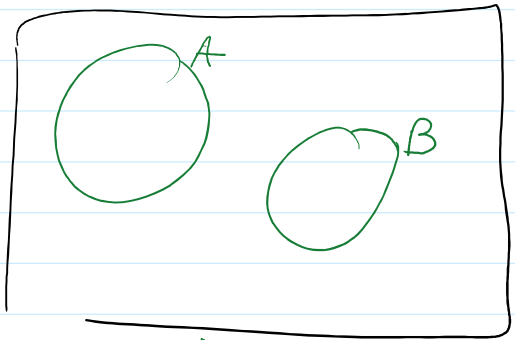


Defn: Disjoint

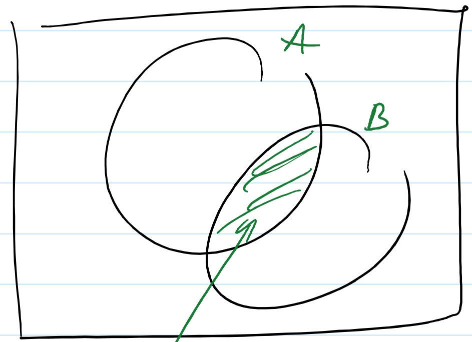
The sets don't overlap.

We say $A, B \subset S$ are disjoint if

$$AB = \emptyset.$$



A disjoint w/ B



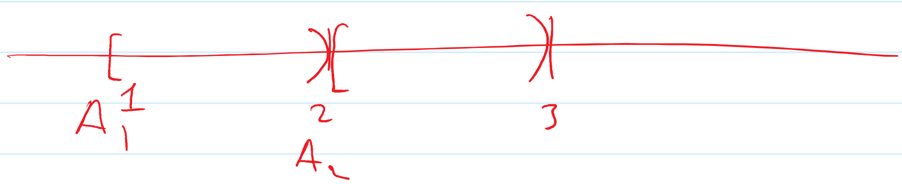
intersection non-empty so A not disjoint w/ B.

Defn: Pairwise Disjoint

A seq. $\{A_i\}_{i=1}^{\infty}$ is pairwise disjoint if

$$A_i A_j = \emptyset \quad \forall\ i \neq j.$$

Ex. $A_i = [i, i+1)$



$$A_i A_j = [i, i+1) \cap [j, j+1) = \emptyset \quad \forall\ i \neq j.$$

So $\{A_i\}_{i=1}^{\infty}$ is pairwise disjoint.
