Lecture 1 - Basic Set Notation

Thursday, January 23, 2020 9:40 AM

Probability

1	2	3	4	5	6	7	8	9	10
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Defn: Set

A set is a collection of objects/things.

Ex.

$$S = \{1, 2, 3\}$$

$$\mathbb{N} = \{1, 2, 3, 4, 5, \ldots\}$$ = natral/counting numbers

$$\mathbb{Q} = \left\{ \frac{m}{n} \text{ when } m, n \text{ are in } \mathbb{N} \right\}$$

(kind rational numbers)

Defn: Set Membership

We say "x is in S", denote,

$$x \in S$$

if S contains x as one of its elements.

Ex. $5 \in \mathbb{N}$

here!

$\{1, 2, 3, 4, 5, 6, 7, \ldots\}$

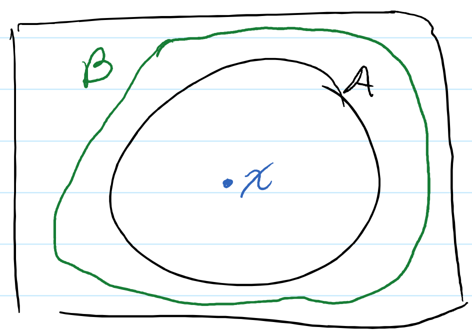
$2/3 \in \mathbb{Q}$

$2/3 \notin \mathbb{N}$ "not in"

<u>Defn: Conteinment</u> (subsets).

We say "A is a subset of B", denoted,

$$A \subset B$$



if $x \in A \Rightarrow x \in B$

implies

<u>ex.</u> $\{1,2,3\} \subset \mathbb{N}$

$\mathbb{Q} \subset \mathbb{R}$ real numbers

$\mathbb{N} \not\subset \{1,2,3\}$

not a subset

<u>Defn: Set Equality</u>

We say "A is equal to B" if

$$A \subset B \text{ \underline{and} } B \subset A.$$

We write $A = B$.

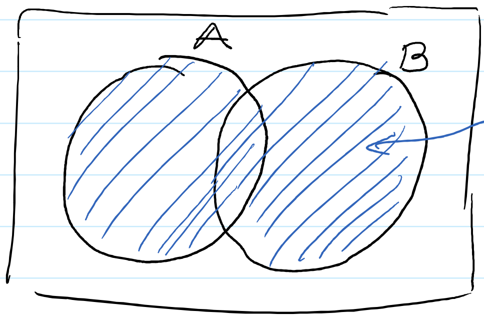
Set Operations

Defn: Union

The union of A and B, denoted $A \cup B$, is defined as

such that

$$A \cup B = \{x \mid x \in A \ \underline{\text{or}} \ x \in B\}$$



Ex. $A = \{1, 2, 3, 4, 5, \ldots\}$

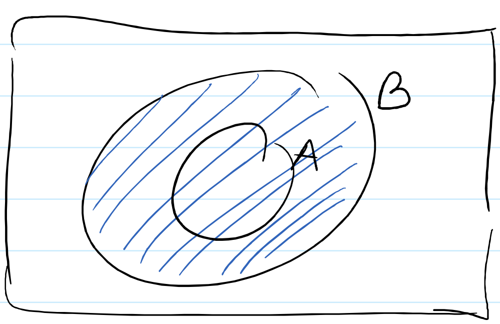
$B = \{-1, -2, -3, -4, \ldots\}$

then $A \cup B = \{\pm 1, \pm 2, \pm 3, \ldots\}$

Ex. $\mathbb{Q} \cup \mathbb{N} = \mathbb{Q}$

b/c $\mathbb{N} \subset \mathbb{Q}$

Fact: if $A \subset B$ then $A \cup B = B$



Ex. $\mathbb{N} \cup \mathbb{N} = \mathbb{N}$

Fact: $A \cup A = A$ (Idempotency)

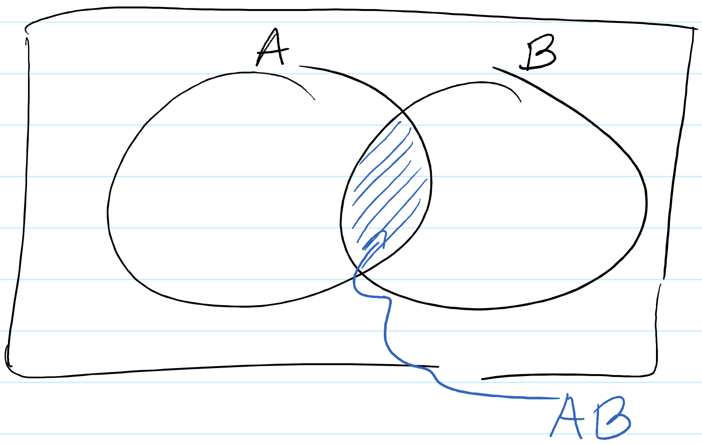
Defn: Intersection

We say the intersection of A and B, denoted,

$$A \cap B \quad \text{or} \quad AB$$

is defined as

$$A \cap B = AB = \{x \mid x \in A \text{ and } x \in B\}$$



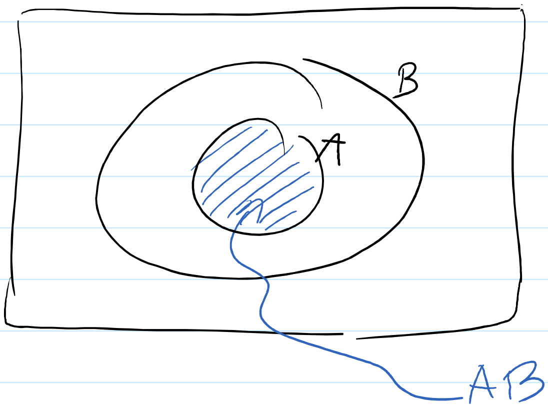
Ex: $A = \{1, 2, 3, 4, \ldots\}$

$B = \{-1, -2, -3, \ldots\}$

then $AB = \emptyset$ empty set

Ex. $\mathbb{Q}\mathbb{N} = \mathbb{N}$

Fact: If $A \subset B$ then $AB = A$



Fact: $AA = A$.

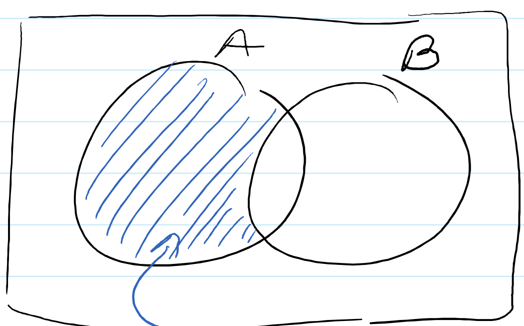
Defn: Set Difference

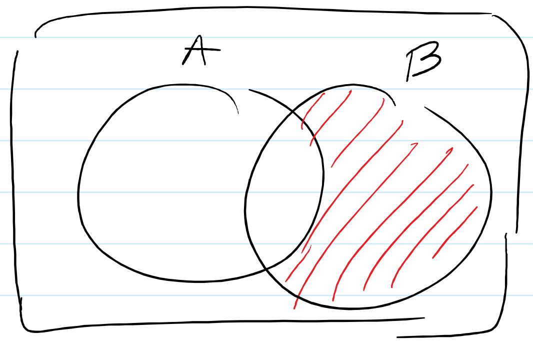
The "difference" between A and B, denoted,

$$A \setminus B$$

is defined as

$$A \setminus B = \{x \mid x \in A \text{ and } x \notin B\}$$





$A \setminus B \neq B \setminus A$

Ex: $A = \{1, 2, 3\}$

$B = \{3, 4, 5\}$

then $A \setminus B = \{1, 2\}$

$B \setminus A = \{4, 5\}$.

Defn: Complements

Need: Some "universe" S in which our sets live.

i.e. $A \subset S$.

Then, the complement of A, denoted A^c, is defined as

$$A^c = \{x \mid x \notin A\}$$
$$= \{x \in S \mid x \in A\}$$

Ex. $A = \{5, 6\}$, $S = \mathbb{N}$ note $A \subset S$

then $A^c = \{1, 2, 3, 4, 7, 8, \ldots\}$

consider: $S = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$

then $A^c = \{0, \pm 1, \pm 2, \pm 3, \pm 4, -5, -6, \pm 7, \ldots\}$

best defn is

$$A^c = S \setminus A.$$

Theorems about Set Operations

① Commutivity: $A \cup B = B \cup A$

$AB = BA$

② Associativity: $A \cup (B \cup C) = (A \cup B) \cup C = A \cup B \cup C$

$A(BC) = (AB)C = ABC$

③ Distributivity: $A(B \cup C) = AB \cup AC$

$A \cup (BC) = (A \cup B)(A \cup C)$

④ De Morgan's Laws:

(i) $(A \cup B)^c = A^c B^c$

(ii) $(AB)^c = A^c \cup B^c$.

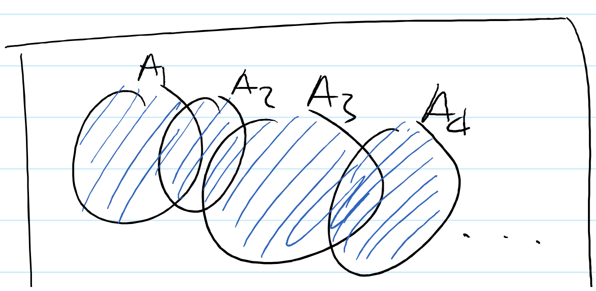
Cantably Infinite Set Operations

Let $A_1, A_2, A_3, A_4, \ldots$ of sets where $A_i \subset S$ for $i = 1, 2, 3 \ldots$

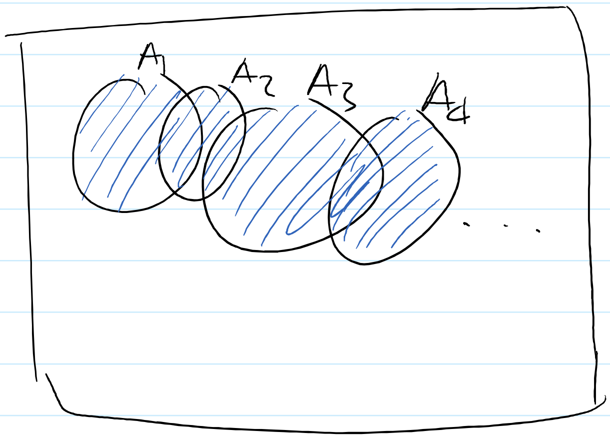
Defn: The union of $\{A_i\}_{i=1}^{\infty}$ denoted

$$\bigcup_{i=1}^{\infty} A_i \overset{def}{=} \{x \in S \mid x \in A_i \text{ for some } i\}.$$

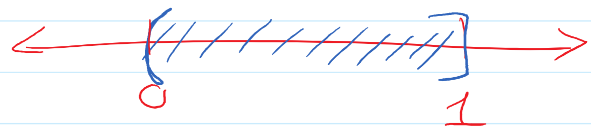
x is in at least one A_i



A_i

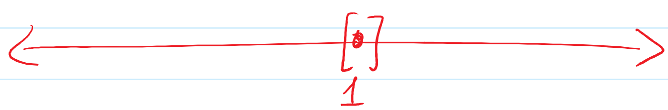


Ex. let $S = (0, 1]$

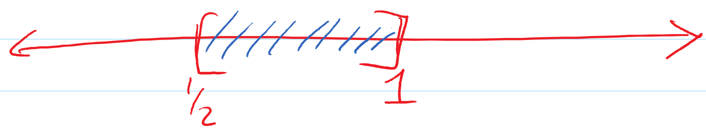


and let $A_i = [1/i, 1]$ for $i = 1, 2, 3, \ldots$

$\{1\} = A_1$



$[1/2, 1] = A_2$



Question: $\bigcup_{i=1}^{\infty} A_i = ? = (0, 1] = S$

includes: 1, $A_2 = [1/2, 1]$

.9

.3? $A_4 = [1/4, 1]$

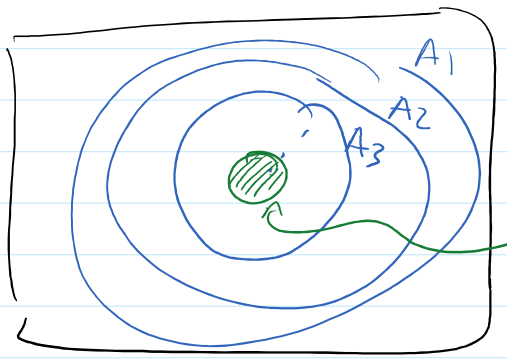
Defn: Infinite Intersection

The intersection $\{A_i\}_{i=1}^{\infty}$ is defined as

$$\bigcap^{\infty} A_i = \{x \in S \mid x \in A_i \text{ for all } i\}$$

$i=1$

$\uparrow$ x is in every A_i



$$\bigcap_{i=1}^{\infty} A_i$$

__Ex.__ $A_i = [1/i, 1]$ then

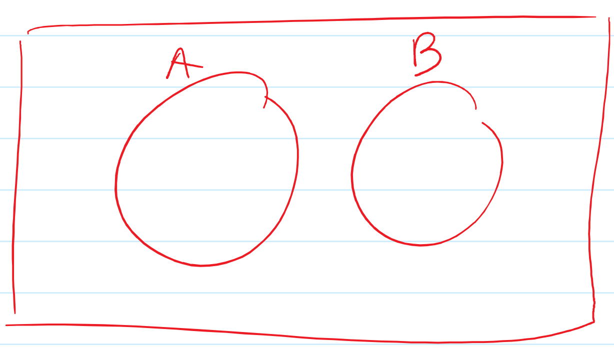
$$\bigcap_{i=1}^{\infty} A_i = \{1\}$$

__Defn! Disjoint__

We say $A, B \subset S$ are __disjoint__ if

$$AB = \varnothing.$$

They share no common elements.



$\leftarrow$ don't overlap.

__Ex.__

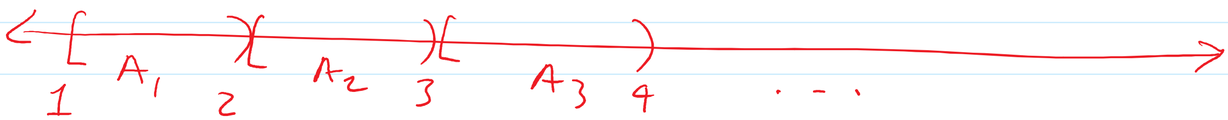
$$\left.\begin{array}{l} A = \{1,2,3\} \\ B = \{4,5,6\} \end{array}\right\} \text{ disjoint b/c } AB = \varnothing.$$

Defn: Pairwise Disjoint

We say a sequence $\{A_i\}_{i=1}^{\infty}$ is **pairwise disjoint** if

$$A_i A_j = \emptyset \text{ for } i \neq j.$$

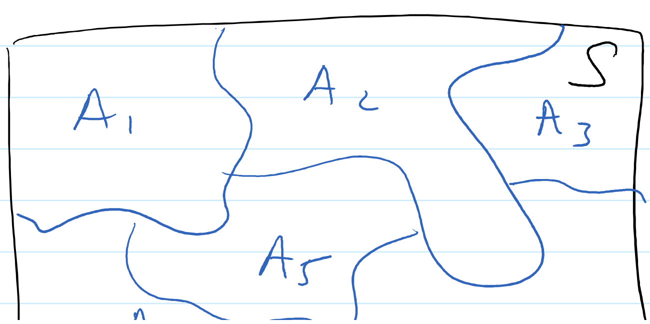
Ex. $A_i = [i, i+1)$ ← these are pairwise disjoint.



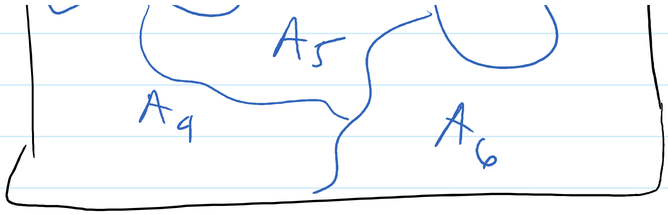
Defn: Partition

Let $\{A_i\}_{i=1}^{\infty}$ a seq. of sets, $A_i \subset S$. We say the collection $\{A_i\}$ is a partition of S if

1. $\{A_i\}$ are pairwise disjoint
2. $\bigcup_{i=1}^{\infty} A_i = S$.



← partition



partition

Ex.

$$A_i = [i, i+1) \subset [1, \infty) = S$$

these $\{A_i\}$ partition $[1, \infty)$.

Defn: Power Set

The power set is the set of all subsets.

For a set A,

$$2^A = \mathcal{P}(A) \overset{\text{def}}{=} \{ B \mid B \subset A \}.$$

Ex. $A = \{1, 2\}$.

$$2^A = \mathcal{P}(A) = \{ \{1\}, \{2\}, \{1,2\}, \emptyset \}$$

notice: $$|2^A| = 2^{|A|}$$

$|\cdot|$ = size of
= # elements.
