Lecture : Set Notation

Defn: Set

A set is a collection of objects.

Ex. $S = \{1, 2, 3\}$

$\mathbb{N} = \{1, 2, 3, \ldots\}$ = natural numbers

$\mathbb{Q} = \{ m/n : m, n \in \mathbb{N}\}$

Defn: Set Membership

We say "x is in S" denoted

$$x \in S$$

if S contains x as an element.

Ex. $5 \in \mathbb{N}$

$2/3 \in \mathbb{Q}$

$2/3 \notin \mathbb{N}$

Defn: Containment

We say "A is a subset of B" denoted $A \subset B$

if $x \in A \Rightarrow x \in B$.

Ex. $\{1,2,3\} \subset \mathbb{N}$

$\mathbb{Q} \subset \mathbb{R}$ = real numbers

$\mathbb{N} \not\subset \{1,2,3\}$

Defn: Set Equality

We say A equals B, denoted

$$A = B$$

if $A \subset B$ and $B \subset A$.

$\boxed{\text{Set Operations}}$

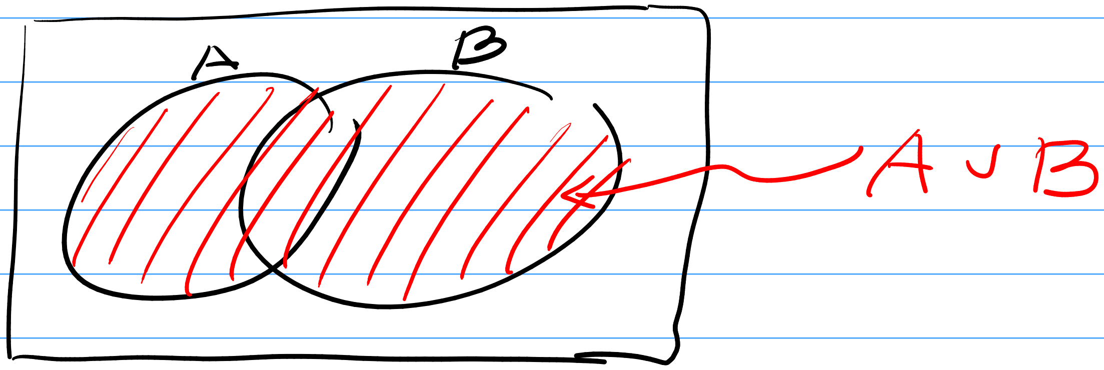
Defn: Union

The union of sets A and B, denoted

$$A \cup B$$

is defined as

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$



Ex. $A = \mathbb{N}$ and $B = \{-1, -2, -3, \dots\}$

then

$$A \cup B = \{\pm 1, \pm 2, \pm 3, \dots\}$$

Ex. $\mathbb{Q} \cup \mathbb{R} = \mathbb{R}$ b/c $\mathbb{Q} \subset \mathbb{R}$

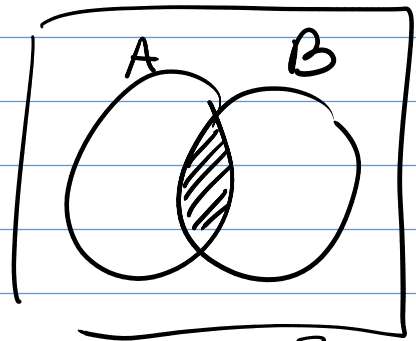
Fact: $A \subset B$ then $A \cup B = B$

Fact: $A \cup A = A$

Defn: Intersection

The intersection of A and B, denoted

$$A \cap B \quad \text{or} \quad AB$$



is defined as

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

Ex. $A = \mathbb{N}$, $B = \{-1, -2, \dots\}$

then $A \cap B = \emptyset$ ← empty set

Ex. $\mathbb{Q}\mathbb{N} = \mathbb{N}$ b/c $\mathbb{N} \subset \mathbb{Q}$

Fact: If $A \subset B$ then $AB = A$

Fact: $AA = A$

Notation: Subset: $\subset$ or $\subseteq$

proper subset: $\subsetneq$

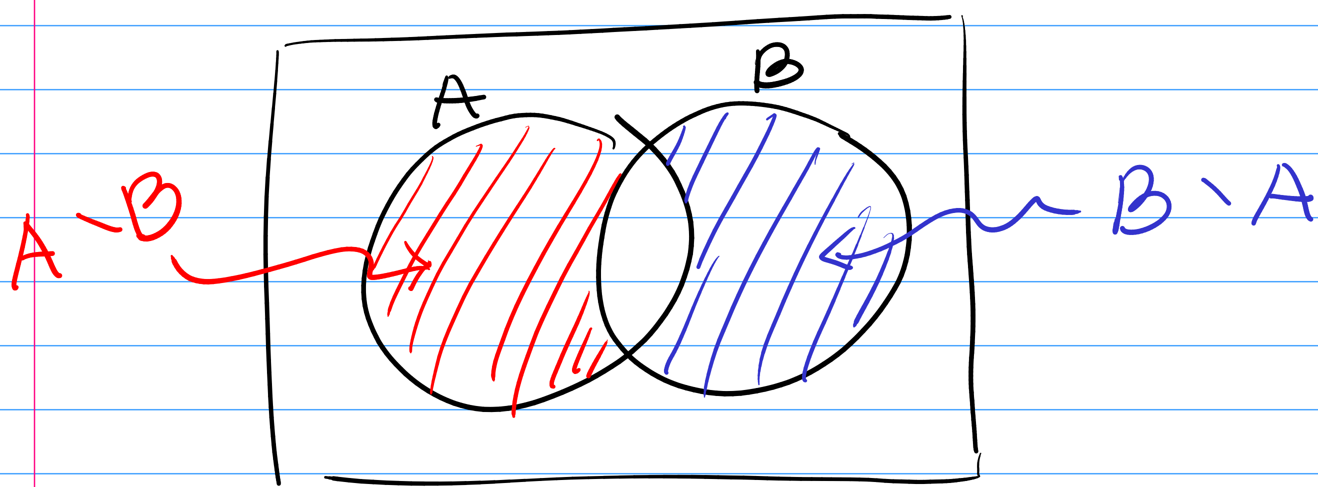
Defn: Set Difference

We say the difference between A and B, denoted

$$A \setminus B$$

is defined as

$$A \setminus B = \{x \mid x \in A \text{ but } x \notin B\}$$



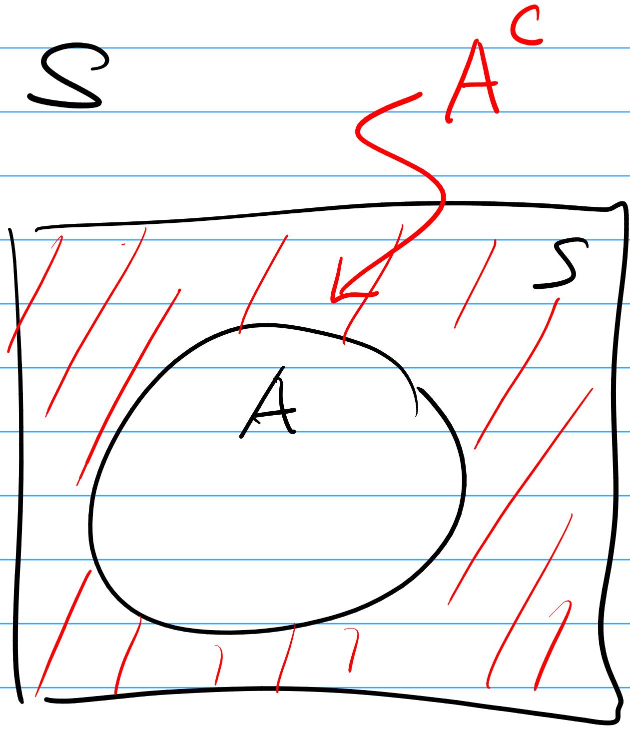
Ex. $A = \{1, 2, 3\}$
$B = \{3, 4, 5\}$ then $A \setminus B = \{1, 2\}$

$B \setminus A = \{4, 5\}$

Defn: Set Complement

Want: $A^c = \{x \mid x \notin A\}$

Need: universe set S



$$A^c = \{x \in S \mid x \notin A\}$$
$$= S \setminus A$$

Ex. $A = \{1, 2\}$, $S = \mathbb{N}$

$$A^c = \{3, 4, 5, \ldots\}$$

Basic Theorems

(1) commuitivity: $A \cup B = B \cup A$

$AB = BA$

(2) Associativity: $A \cup (B \cup C) = (A \cup B) \cup C$

$$A(BC) = (AB)C$$

(3) Distributivity:

$$A \cup (BC) = (A \cup B)(A \cup C)$$

$$A(B \cup C) = AB \cup AC$$

(4) De Morgan's Laws:

(I) $(A \cup B)^c = A^c B^c$

(II) $(AB)^c = A^c \cup B^c$

Countably Infinite Set Operations

Let $A_1, A_2, A_3, \ldots$ be a countable collection of subsets of S.

notation: $(A_i)_{i=1}^{\infty}$

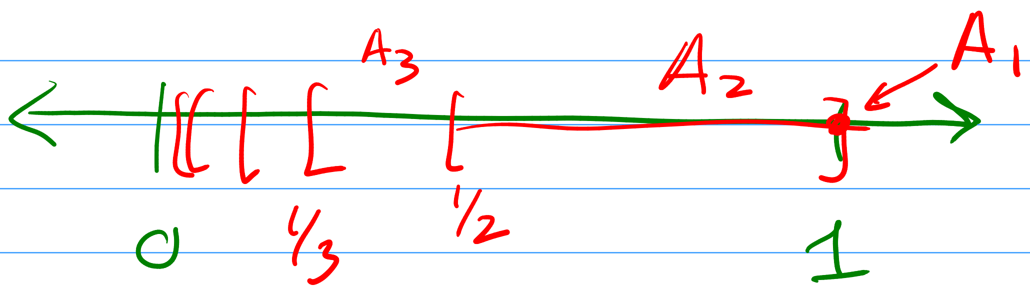
Defn: Countable Union

$$\bigcup_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \text{ for some } i\}$$

Ex. $S = (0, 1]$

$A_i = [1/i, 1]$

$$\bigcup_{i=1}^{\infty} A_i = (0, 1] = S$$



Defn: Countable Intersection

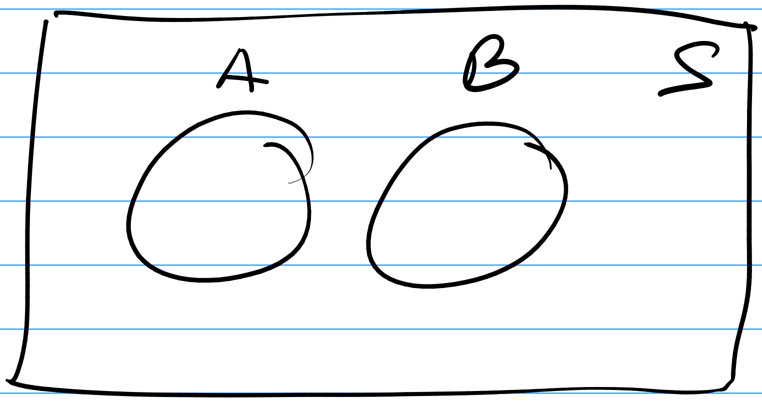
$$\bigcap_{i=1}^{\infty} A_i = \{x \in S \mid x \in A_i \text{ for all } i\}$$

Ex. (from prev.)

$$\bigcap_{i=1}^{\infty} A_i = \{1\}$$

Defn: Disjoint

We say that A and B are disjoint if $AB = \emptyset$.



$AB = \emptyset$

Ex. $A = \{1, 2, 3\}$
$B = \{4, 5, 6\}$

then $AB = \emptyset$ so they are disjoint.

Defn: Pairwise Disjoint

A seq (A_i) is pairwise disjoint if
$$A_iA_j = \emptyset \text{ for } i \neq j.$$

Ex. If $A_i = [i, i+1)$ for $i = 1, 2, 3, \ldots$

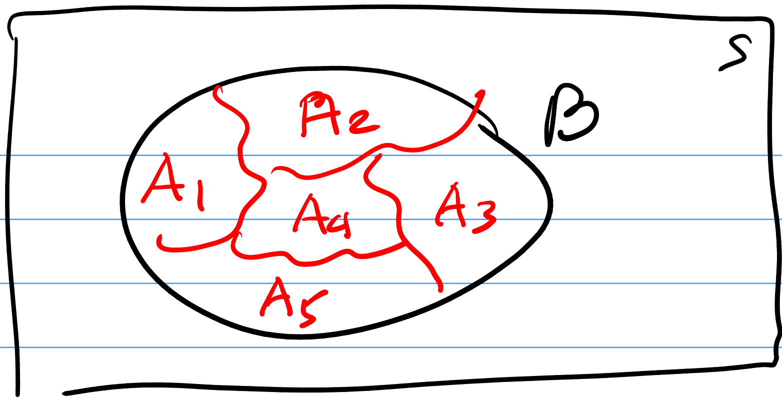
then $$A_iA_j = \emptyset \quad \forall i \neq j$$

Defn: Partition

We say a sequence (A_i), where $A_i \subset B$, partitions B if

I) the A_i are (pairwise) disjoint

II) $\bigcup_i A_i = B$



Ex. $A_i = [i, i+1)$
partition $[1, \infty)$

Defn: Power Set

The power set of a set A is the set of all subsets of A

notation: $\mathcal{P}(A)$ or 2^A

mathy:

$$2^A = \{ B \mid B \subset A \}$$

Ex.

$A = \{1, 2\}$ then

$2^A = \{\{1\}, \{2\}, \{1,2\}, \emptyset\}$

<u>Fact:</u> $|2^A| = 2^{|A|}$

$|\cdot| =$ card (size) of $\cdot$