SP 3.7

4A, 5B, 6C $\quad$ $\boxed{n = 15 = 4+5+6}$

$\boxed{r=3}$ sampled w/o repl.

Prob. $\boxed{\text{all different}}$

E

S ← all equally likely

Do in ordered way.

$$\boxed{|S| = \frac{15!}{12!} = 15 \cdot 14 \cdot 13}$$

	ordered	unordered
w/ repl.	n^r	$\binom{n+r-1}{r}$
w/o repl.	$\frac{n!}{(n-r)!}$	$\binom{n}{r}$

r!

$S = \{(a_2, b_3, c_1), (a_1, a_2, b_3), \dots \}$

(b_3, a_2, c_1),

$a_1, a_2, \dots, a_4$
$b_1, \dots, b_5$
$c_1, \dots, c_6$

$$\boxed{E = \{(a_1, b_2, c_3), (c_3, b_2, a_1), (a_2, b_1, c_4), \ldots\}}$$

$|E| =$ 1 choose which a

2 " b

3 " c

4 order them

$$|E| = 4 \cdot 5 \cdot 6 \cdot 3!$$

$$\boxed{\text{So } P(E) = \frac{4 \cdot 5 \cdot 6 \cdot 3!}{15 \cdot 14 \cdot 13}}$$

SP 3.8

$$P(E) = \frac{|E|}{|S|} = \frac{4 \cdot 5 \cdot 6 \cdot 3!}{15^3}$$

Modify: Sample 4

want: 2 A, 1 B, 1 C

$$E = \{ \underbrace{(a_1, a_2, b_1, c_3)}_{\text{w/o repl.}}, (a_2, a_1, b_1, c_3), \ldots \}$$

$$|E| = \boxed{\binom{4}{2}\binom{5}{1}\binom{6}{1} \cdot 4!} = \boxed{4 \cdot 3 \cdot 5 \cdot 6 \cdot 4!} / 2!$$

(a_1, a_2, b_1, c_1)

(a_2, a_1, b_1, c_1)

w/ repl:

(a_1, a_1, b_1, c_1)

$4 \cdot 4 \cdot 5 \cdot 6$

6 red, 4 blue, 5 green $n=15$

P(3 red, 1 blue, 1 green) $r=5$

(a) w/ repl. [ordered]

$|S| = 15^5$

$r_1 r_3 r_4$
$r_4 r_3 r_1$

$|E| = 6 \cdot 6 \cdot 6 \cdot 4 \cdot 5 \cdot 5!/3!$

$$P(E) = \frac{6^3 \cdot 4 \cdot 5 \cdot 5!/3!}{15^5}$$

$$f(k) = \frac{1}{e}\frac{1}{k!} \quad \text{for } k = 0, 1, 2, 3, \ldots$$

(a) $$E[X] = \sum_{k=0}^{\infty} k f(k)$$

$$(*) = \sum_{k=\cancel{0}\,1}^{\infty} k \frac{1}{e} \cdot \frac{1}{k!}$$

$$= \frac{1}{e} \sum_{k=1}^{\infty} 1/(k-1)!$$

$\underline{\text{hint}}$: $e = \sum_{k=0}^{\infty} 1/k!$

$$= \frac{1}{e} \sum_{k=0}^{\infty} 1/k!$$

$$= \frac{1}{e} e = 1$$

(b) $$E[X(X-1)] = \sum_{k=0}^{\infty} k(k-1) f(k)$$

$$= \sum_{k=2}^{\infty} k(k-1) \frac{1}{e} \frac{1}{k!}$$

$$= \sum_{k=2}^{\infty} \frac{1}{e} \frac{1}{(k-2)!}$$

$$= 1$$

(c) $Var(X) = E[X^2] - E[X]^2$

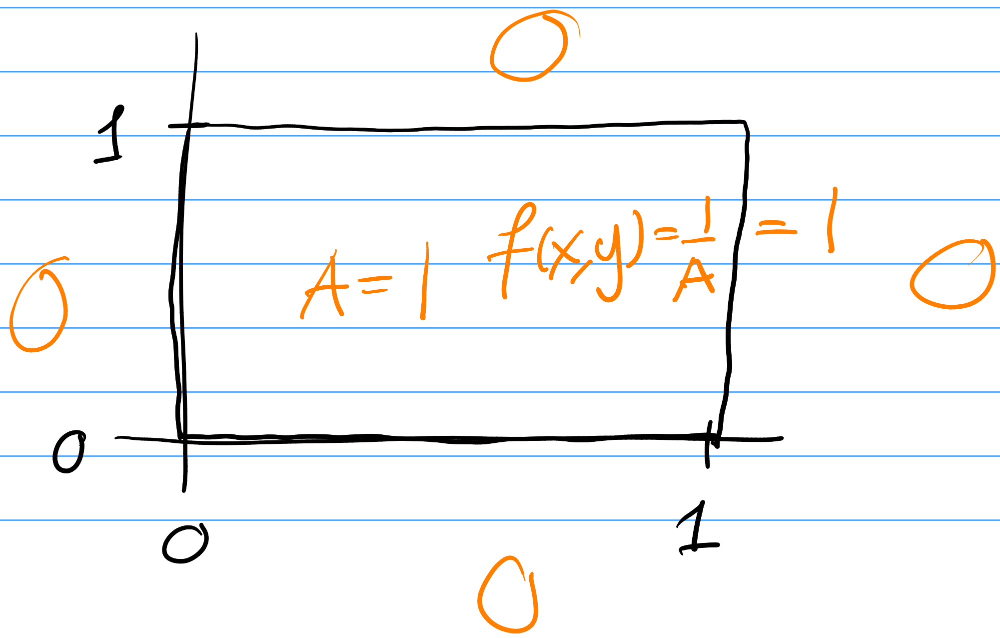
$E[X(X-1)] = 1$

$\hookrightarrow E[X^2 - X] = E[X^2] - E[X]$

$= E[X^2] - 1 = 1$

$E[X^2] = 2$

X, Y unif. over unit square.



① Joint

$$f(x,y) = 1 \quad \text{for} \quad \begin{matrix} 0 < x < 1 \\ 0 < y < 1 \end{matrix}$$

$$\boxed{U = X + Y \,,\, V = X - Y}$$

② Inverses

$$\boxed{x = g_1^{-1}(u,v) = \frac{1}{2}(u+v)}$$

$$u + v = 2x$$

$$\Rightarrow x = \frac{1}{2}(u+v)$$

$$\boxed{y = g_2^{-1}(u,v) = \frac{1}{2}(u-v)}$$

$$u - v = 2y \Rightarrow y = \frac{1}{2}(u-v)$$

③

$$J = \begin{bmatrix} \frac{\partial g_1^{-1}}{\partial u} & \frac{\partial g_1^{-1}}{\partial v} \\ \frac{\partial g_2^{-1}}{\partial u} & \frac{\partial g_2^{-1}}{\partial v} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$$

$$|\det J| = \left| \left(\frac{1}{2}\right)\left(\frac{-1}{2}\right) - \left(\frac{1}{2}\right)\left(\frac{1}{2}\right) \right| = \left| -\frac{1}{2} \right| = \frac{1}{2}$$

4

$$f(u,v) = f_{X,Y}(g_1^{-1}(u,v), g_2^{-1}(u,v))\,|\det J|$$

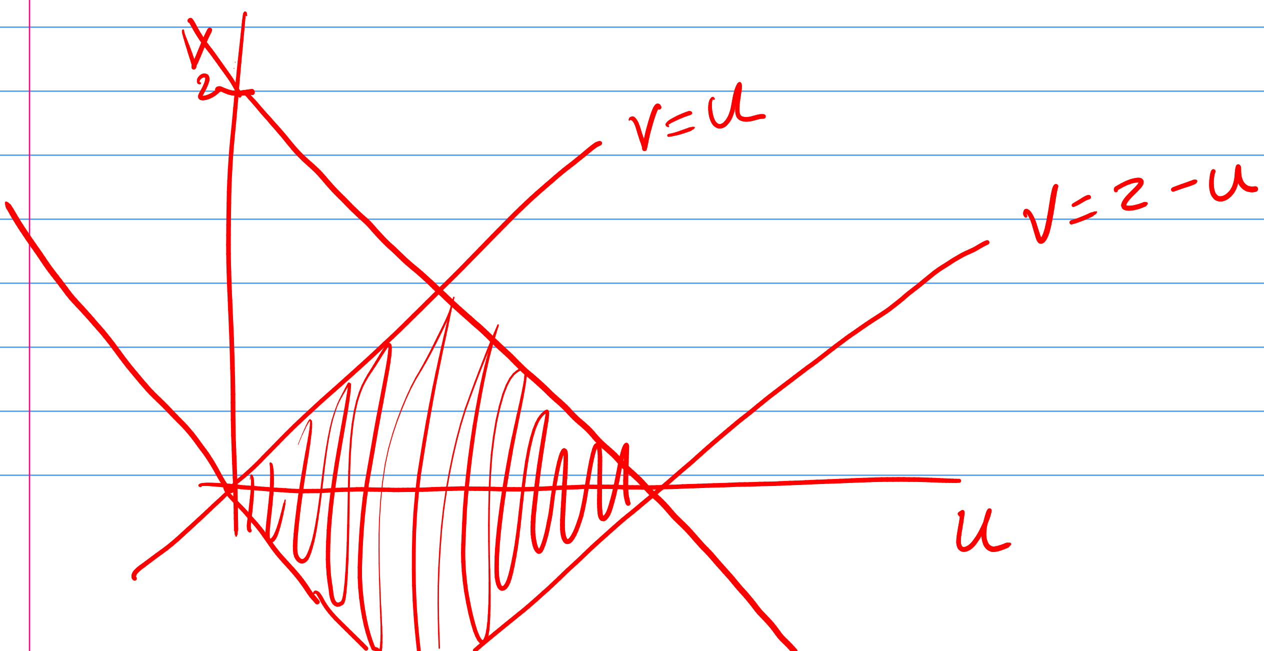
$$= f_{X,Y}\left(\tfrac{1}{2}(u+v), \tfrac{1}{2}(u-v)\right)\frac{1}{2}$$

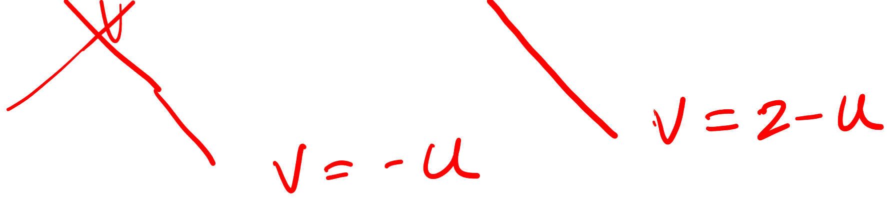
$$\boxed{= 1 \cdot \frac{1}{2} \quad \text{for } \boxed{\begin{array}{l} 0 < \frac{1}{2}(u+v) < 1 \\ 0 < \frac{1}{2}(u-v) < 1 \end{array}}}$$

$0 < u+v < 2 \Leftrightarrow \boxed{-u < v < 2-u}$

$0 < u-v < 2 \Leftrightarrow -u < -v < 2-u$

$\boxed{u-2 < v < u}$





X cts, $F_X(X) \sim U(0,1)$

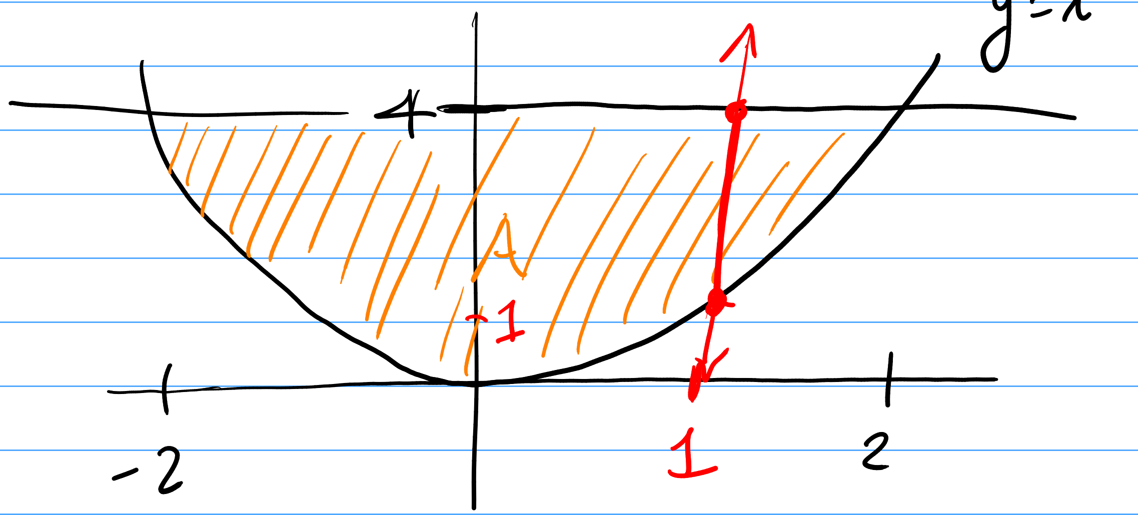
$X \sim F_X$

$U = F_X(X) \sim U(0,1)$

$\boxed{F_X^{-1}(\underline{U})} = F_X^{-1}F_X(X) = X \sim F_X$

$y = F(x) = 1 - e^{-\lambda x}$

$A = \{(x,y),\ 0 < x^2 < y < 4\}$



$$E[Y|X=1] = \int y\, f(y|1)\, dy$$

$$f(y|x) = \frac{f(x,y)}{f_X(1)} = \frac{1/A}{3/A} = \frac{1}{3}$$

$$f_X(1) = \int_1^4 f(1,y)\, dy = \frac{3}{A}$$
