Midterm 1 Review

Thursday, September 26, 2024 11:01 AM

SP1 # 10 $X_n \overset{iid}{\sim} \text{Pois}(\lambda)$

What is the MGF of $\bar{X}$?

$$M_{\bar{X}}(t) = M(t/N)^N \quad \leftarrow N = \text{sample size.}$$

M = MGF of any X_n

$$M(t) = \exp(\lambda(e^t - 1))$$

$$\boxed{M_{\bar{X}}(t) = \exp(\lambda(e^{t/N} - 1))^N}$$

SP2 # 10

$X_n \overset{iid}{\sim} f_\lambda$ where

$$f_\lambda(x) = \frac{1}{2}\lambda^3 x^3 \exp(-\lambda x) \text{ for } x > 0$$

Use factorization thm to find a 1-D suff. stat. for λ.

Fact Thm: If

$$f(\underline{x}) = g(\lambda, T)\, h(\underline{x})$$

then T is sufficient for λ.

$$f(\underline{x}) = \prod_{n=1}^{N} \frac{1}{2}\lambda^3 x_n^3 \exp(-\lambda x_n)\, \mathbb{1}(x_n > 0)$$

$$= \left(\frac{1}{2}\right)^N \lambda^{3N} \left(\prod_n x_n^3\right) \exp\left(-\lambda \overbrace{\sum_n x_n}^{N\bar{x}}\right) \prod_n \mathbb{1}(x_n > 0)$$

$$= \underbrace{\left(\frac{1}{2}\right)^N \lambda^{3N} \exp(-\lambda N \bar{x})}_{g(\lambda, T)} \underbrace{\left(\prod_n x_n^3\right)\left(\prod_n \mathbb{1}(x_n > 0)\right)}_{h(\underline{x})}$$

$$T = \bar{x}$$

By Fact. thm $\bar{X}$ suff. for λ.

SP1#4

iid

$$\left(\frac{1}{2}\right)^{n} \cdot \binom{n}{2}$$

$$\ell(\mu,\lambda) = \frac{N}{2}\log\frac{\lambda}{2\pi} \cdots$$

$$-\frac{\lambda}{2\mu}\sum_n \frac{(x_n-\mu)^2}{x_n}$$

$$\propto \frac{N}{2}\log(\lambda) - \frac{\lambda}{2\mu}\sum_n \frac{x_n^2 - 2\mu x_n + \mu^2}{x_n}$$

$$\propto \frac{N}{2}\log(\lambda) - \frac{\lambda}{2\mu}\left[\sum_n x_n - 2\mu N + \sum_n \frac{\mu^2}{x_n}\right]$$

$$\ell(\mu,\lambda) \propto \frac{N}{2}\log(\lambda) - \frac{\lambda}{2}\left[\frac{\sum_n x_n}{\mu} - 2N + \sum_n \frac{\mu}{x_n}\right]$$

$$\frac{\partial \ell}{\partial \mu} = 0 \quad \text{and} \quad \frac{\partial \ell}{\partial \lambda} = 0$$

$$-2N + \sum_n \frac{\mu}{x_n}\Big] = 0$$

$$\frac{\partial \ell}{\partial \lambda} = \frac{N}{2}\frac{1}{\lambda} - \frac{1}{2}\left[N\bar{x} \quad \checkmark \quad \frac{1}{N}\sum\right.$$

$$\hat{\lambda} = \left[\frac{\bar{x}}{\hat{\mu}} - 2 + \frac{1}{N}\sum_n \frac{\hat{\mu}}{x_n}\right]^{-1}$$

$$\frac{\partial \ell}{\partial} = -\frac{\lambda}{2}\left[-\frac{\sum x_n}{\cdot} + \sum \frac{1}{x_i}\right] = 0$$

$$\frac{\partial \ell}{\partial \mu} = \frac{n}{2}\left[-\frac{\sum_n x_n}{\mu^2} + \sum \frac{1}{x_n}\right] = 0$$

$$\sum \frac{1}{x_n} = \frac{\sum_n x_n}{\mu^2}$$

$$\hat{\mu} = \sqrt{\frac{\sum_n x_n}{\sum_n 1/x_n}}$$

Sp4 #8 $X_n \overset{iid}{\sim} U(-\theta, \theta)$. Find MLE.

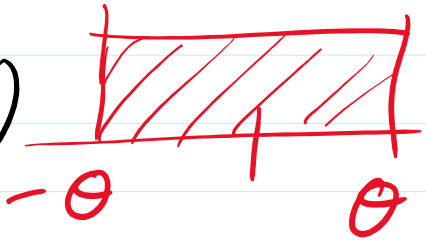
$$L(\theta) = \prod_n \frac{1}{2\theta} \mathbb{1}(|x_n| < \theta)$$

$$f(x_n) = \frac{1}{2\theta} \cdot \mathbb{1}(-\theta < x_n < \theta)$$

$$\mathbb{1}(|x_n| < \theta)$$

$$= 2^{-N} \theta^{-N} \prod_n \mathbb{1}(|x_n| < \theta)$$

$$L(\theta) = 2^{-N} \theta^{-N} \mathbb{1}(\underbrace{\max_n |x_n|}_{T} < \theta)$$



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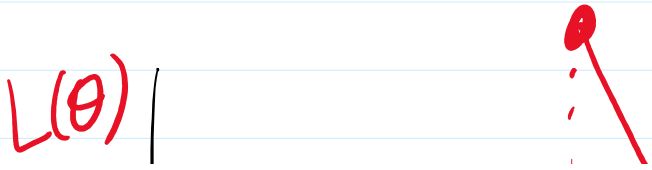

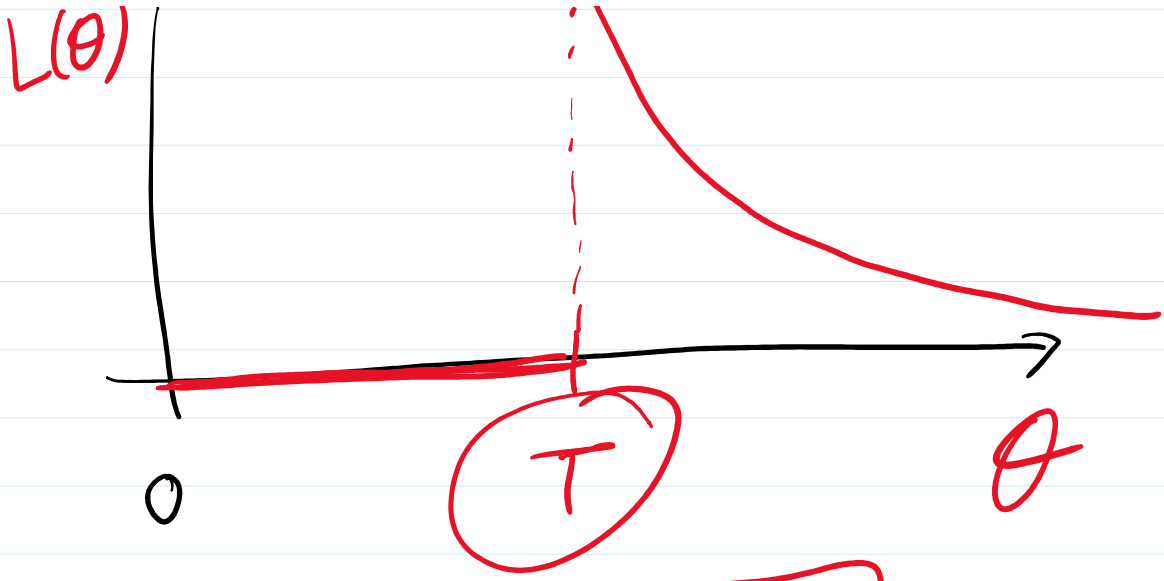

$$\hat{\theta}_{MLE} = \max_n |X_n|.$$