Suggested Problems 1

(1) Let $X_n \overset{iid}{\sim} Bernoulli(p)$. What is the PMF of $G = \left(\prod_{i=n}^{N} X_n\right)^{\frac{1}{n}}$?

(2) Let $X_1 \sim Bin(10, 1/8)$ and $X_2 \sim Bin(4, 1/8)$ and $X_3 \sim Bin(6, 1/8)$ be mutually independent. Find a formula for $P(\bar{X} < 1.8)$.

(3) Let X_1 and X_2 be i.i.d from $U(0, 1)$. Find $P(\bar{X} > 0.8)$.

(4) Let X_1, X_2, X_3 be i.i.d from $Bernoulli(p)$. Find the PMF of $\bar{X}$.

(5) Let X_1 and X_2 have joint density

$$f(x_1, x_2) = 2 \text{ for } 0 < x_1 < x_2 < 1.$$

Find the PDF of $\bar{X}$.

(6) Let $X_n \overset{iid}{\sim} U(0, 1)$. Show that $X_{(1)} \sim Beta(1, N)$.

(7) Let $X_n \overset{iid}{\sim} U(0, 1)$. Show that $X_{(N)} \sim Beta(N, 1)$.

(8) Let $X_n \overset{iid}{\sim} Exp(\lambda)$. Show that $X_{(1)} \sim Exp(N\lambda)$.

(9) Let $X_n \overset{iid}{\sim} Exp(\lambda)$. Show that $\bar{X} \sim Gamma(N, N\lambda)$.

(10) Let $X_n \overset{iid}{\sim} Poisson(\lambda)$. What is the MGF of $\bar{X}$?