Suggested Problems 10

(1) Let $X_n \overset{iid}{\sim} f_\theta$ where
$$f_\theta(x) = \frac{\theta}{x^{\theta+1}} \text{ for } x > 1.$$
Find the LRT test statistic for testing $H_0 : \theta = \theta_0$ v. $H_a : \theta \neq \theta_0$.

(2) Let $X_n \overset{iid}{\sim} f_\theta$ where
$$f_\theta(x) = \frac{1}{\theta} \text{ for } x = 1, 2, 3, \dots, \theta.$$

   (a) Find the LRT test statistic for testing $H_0 : \theta = 20$ v. $H_a : \theta < 20$.
   (b) Find the critical value so that for $N = 1$ observation the test has level $\alpha = 0.05$.
   (c) Draw a plot of the power function for θ when $N = 1$.

(3) Let $X_n \overset{iid}{\sim} Pois(\lambda)$. Find the LRT test statistic for testing $H_0 : \lambda = \lambda_0$ v. $H_a : \lambda < \lambda_0$.

(4) Let $X_n \overset{iid}{\sim} f_\lambda$ where
$$F_\lambda(x) = 1 - \exp(-(\lambda x)^2) \text{ for } x > 0.$$
Find the LRT test statistic for testing $H_0 : \lambda = \lambda_0$ v. $H_a : \lambda < \lambda_0$.

(5) Let $X_n \overset{iid}{\sim} f_\theta$ where
$$f_\theta(x) = \theta x \text{ for } 0 < x < \sqrt{2/\theta}.$$
Find the LRT test statistic for testing $H_0 : \theta = 8$ v. $H_a : \theta > 8$.