Suggested Problems 11

(1) If $L(x)$ and $U(x)$ satisfy $P(L \leq \theta) = 1 - \alpha_1$ and $P(U \geq \theta) = 1 - \alpha_2$ and $L < U$ show that $P(L < \theta < U) = 1 - \alpha_1 - \alpha_2$.

(2) Let $X_n \overset{iid}{\sim} F$ where

$$F(x) = \begin{cases} 0 & x < 0 \\ (x/\beta)^5 & 0 < x < \beta \\ 1 & x \geq \beta \end{cases}$$

then $T = X_{(N)}$ has the CDF $F_T(t) = (t/\beta)^{5N}$. Use T to construct a one-sided 95% confidence interval for β of the form $(-\infty, U)$.

(3) Let X_n have a shifted exponential distribution so that

$$F(x) = 1 - \exp(-(x - \mu)) \text{ for } x > \mu$$

Then if $T = X_{(1)}$ we have

$$F_T(t) = 1 - \exp(-N(x - \mu)) \text{ for } t > \mu.$$

Use T to construct a $(1 - \alpha)$ confidence interval for μ.

(4) Given $X_n \overset{iid}{\sim} Bern(p)$ derive the form of a $(1 - \alpha)$ confidence interval for p by inverting the LRT test $H_0 : p = p_0$ v $H_a : p \neq p_0$.

(5) Let $X_n \overset{iid}{\sim} f_\mu$ where $f_\mu(x) = g(x - \mu)$ for some function g not involving μ. Show that $\bar{X} - \mu$ is pivotal for μ.

(6) Let $X_n \overset{iid}{\sim} f_\sigma$ where $f_\sigma(x) = \frac{1}{\sigma} g(x/\sigma)$ for some function g not involving σ. Show that $\bar{X}/\sigma$ is pivotal for σ.

(7) Let $X_n \overset{iid}{\sim} N(\theta, \theta)$. Argue that $(\bar{X} - \theta)/\sqrt{\theta}$ is a pivot for θ and use it to find the form of a $(1 - \alpha)$ CI for θ.

(8) Let $X \sim Beta(\theta, 1)$ and let $Y = -1/\log(X)$. Evaluate the confidence coefficient for the CI $[y/2, y]$. Hint: $f(y) = \theta/y^2 \exp(-\theta/y)$ for $y > 0$.

(9) Let $X \sim Beta(\theta, 1)$. Notice that $Y = X^\theta \sim U(0, 1)$. Use Y to construct the form of a $(1 - \alpha)$ CI for θ.

(10) Let $X \sim U(\theta - 1/2, \theta + 1/2)$. Find a $(1 - \alpha)$ CI for θ based on X.