Suggested Problems 2

(1) Let Z_1 and Z_2 be i.i.d from $N(0,1)$. Find $P(Z_1^2 + Z_2^2 < 1)$.

(2) Let $X_1, \ldots, Y_N \overset{iid}{\sim} N(\mu, \sigma^2)$ and $Y_1, \ldots, Y_N \overset{iid}{\sim} N(\mu, \sigma^2)$ be two independent sets of random variables. Find the median of $\bar{X} - \bar{Y}$.

(3) Let X_1, X_2, X_3 be i.i.d from $N(\mu = 60, \sigma^2 = 10)$. Find $E[S^2]$ and $Var(S^2)$.

(4) Let $X_1, \ldots, X_N$ be i.i.d from $N(\mu_X, \sigma^2)$ and $Y_1, \ldots, Y_M$ be i.i.d from $N(\mu_Y, 4\sigma^2)$ and the Xs and Ys are independent. Find the distribution of

$$\frac{4\sum_{n=1}^{N}(X_n - \bar{X})^2 + \sum_{m=1}^{M}(Y_m - \bar{Y})^2}{4\sigma^2}.$$

(5) Let X_1, X_2, X_3, X_4 be i.i.d $N(0,1)$ What is the distribution of

(a) $(X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 + (X_3 - \bar{X})^2 + (X_4 - \bar{X})^2$

(b)

$$\frac{(X_1 - X_2 + X_3 + X_4)^2}{4}$$

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(6) Let $X_n \overset{iid}{\sim} Geometric(p)$ so that $f(x) = p(1-p)^{x-1}$ for $x = 1, 2, 3, \ldots$. Use the factorization theorem to find a sufficient statistic for p.

(7) Let $X_n \overset{iid}{\sim} f_\theta$ where

$$f_\theta(x) = 2x\theta^{-2}\exp(-(x/\theta)^2) \text{ for } x > 0.$$

Use the factorization theorem to find a sufficient statistic for θ.

(8) Let $X_n \overset{iid}{\sim} U(0, \theta)$. Use the factorization theorem to find a sufficient statistic for θ.

(9) Let $X_n \overset{iid}{\sim} f_\theta$ where

$$f_\theta(x) = \frac{\theta}{(1+x)^{\theta+1}} \text{ for } x > 0.$$

Use the factorization theorem to find a sufficient statistic for θ.

(10) Let $X_n \overset{iid}{\sim} f_\lambda$ where

$$f_\lambda(x) = \frac{1}{2}\lambda^3 x^3 \exp(-\lambda x) \text{ for } x > 0.$$

Use the factorization theorem to find a sufficient statistic for λ.

(11) Let $X_n \overset{iid}{\sim} N(\mu, 1)$. Show that the joint distribution of the X_n are an exponential family. Identify $T(\underset{\sim}{x}) = T(x_1, \ldots, x_n)$.

(12) Let $X_n \overset{iid}{\sim} Geometric(p)$ so that $f(x) = p(1-p)^{x-1}$ for $x = 1, 2, 3, \ldots$. Show that the joint distribution of the X_n are an exponential family. Identify $T(\underset{\sim}{x}) = T(x_1, \ldots, x_n)$.