Suggested Problems 4

(1) Let $X_n \overset{iid}{\sim} f_\theta$ where

$$f_\theta(x) = \begin{cases} \theta & x = -1 \\ 2\theta & x = 0 \\ 1 - 3\theta & x = 1. \end{cases}$$

Is the MOM estimator of θ unbiased?

(2) Let $X_n \overset{iid}{\sim} f_\theta$ where

$$f_\theta(x) = \theta(x - 1/2) + 1 \text{ for } 0 < x < 1.$$

Is the MOM estimator of θ unbiased?

(3) Let $X_n \overset{iid}{\sim} Exp(\lambda)$. Find a constant k so that $kX_{(1)}$ is unbiased for $1/\lambda$.

(4) Let X_1 and X_2 be a i.i.d. random sample from a distribution with mean μ and variance σ^2. Consider $\hat{\mu}_1 = \bar{X}$ and $\hat{\mu}_2 = (2X_1 + X_2)/3$. Which estimator do we prefer?

(5) Let $X_1, \ldots, X_N$ be i.i.d from $N(\mu_X, \theta)$ and $Y_1, \ldots, Y_M$ be i.i.d from $N(\mu_Y, \theta)$ and the Xs and Ys are independent.
  (a) Find the MLEs for μ_x, μ_y and θ.
  (b) Is $\hat{\theta}$ unbiased for θ?

(6) Let $X_n \overset{iid}{\sim} f_\lambda$ where

$$f_\lambda(x) = \left(\frac{\lambda}{2\pi x^3}\right)^{1/2} \exp\left(-\lambda(x - \mu)^2/(2\mu^2 x)\right)$$

Find a the MLEs for μ and λ.

(7) Let $X_n \overset{iid}{\sim} f_\theta$ where

$$f_\theta(x) = \frac{\theta x^{\theta - 1}}{2^\theta} \text{ for } 0 < x < 2.$$

Find the MLE for θ.

(8) Let $X_n \overset{iid}{\sim} U(-\theta, \theta)$. Find the MLE for θ.

(9) Let $X_n \overset{iid}{\sim} U(0, \theta)$.
  (a) Find the MLE for the variance of a $U(0, \theta)$.
  (b) What is the variance of the MLE for θ?

(10) Let $X_n \overset{iid}{\sim} f_\theta$ where

$$f_\theta(x) = \exp(-(x - \theta)) \text{ for } x \geq \theta.$$

What is the MLE for θ?