Suggested Problems 8

(1) Let $X_i \overset{iid}{\sim} Exponential(\lambda)$. Show that $Y_n = \min_{i=1,\dots,n} X_i \overset{p}{\to} 0$.

(2) Let X_n be independent with mean μ and variance $\sigma^2 < \infty$. Show that $\bar{X}_N = \frac{1}{N}\sum_{n=1}^N X_n$ is consistent for μ.

(3) Let $X_n = X + Y_n$ where $\mathbb{E}[Y_n] = 1/n$ and $Var(Y_n) = \sigma^2/n$. Show that $X_n \overset{p}{\to} X$.

(4) Let $X_n \overset{iid}{\sim} Exp(n)$ so that $\mathbb{E}[X_n] = 1/n$. Show that $X_n \overset{p}{\to} 0$.

(5) Let the CDF of X_n be $F_{X_n}(x) = (1 - (1 - \frac{1}{n})^{nx})\mathbb{1}(x > 0)$. Show that $X_n \overset{d}{\to} Exp(1)$.

(6) Let the sample space be $S = \{H, T\}$ be equally likely outcomes of flipping a coin. Define

$$X_n(s) = \begin{cases} \frac{n}{n+1} & s = H \\ (-1)^n & s = T \end{cases}.$$

Does $X_n \overset{a.s.}{\to} 1$?

(7) Let $X_n \overset{iid}{U}(0, \theta)$ and $X_{(N)} = \max_{i=1,\dots,N} X_i$. Show that $X_{(N)} \overset{p}{\to} \theta$.

(8) Let $Y_N \sim Bin(N, p)$. Show that $Y_N/N \overset{p}{\to} p$.

(9) If X_n is a seq. of random variables with MGFs $M_{X_n}(t)$ then if

$$M_{X_n}(t) \overset{n}{\to} M(t)$$

we have that $X_n \overset{d}{\to} X$ where X is a r.v. with MGF $M(t)$.

Let $M_{X_n}(t) = \left(\frac{\lambda}{\lambda - t}\right)^n$. What does X_n/n converge to in distribution?

(10) Let $X_n \overset{p}{\to} X$ for some r.v. X and $Y_n \overset{p}{\to} 5$ and $a_n \overset{n}{\to} 7$. What does $a_n Y_n e^{X_n}$ converge to in probability?