Quiz Problem 1

Let

$$A = \begin{bmatrix} 1 & 1 \\ 2 & -2 \\ 2 & 2 \end{bmatrix}.$$

Note that $A^T A$ has a spectral decomposition

$$A^T A = \begin{bmatrix} 9 & 1 \\ 1 & 9 \end{bmatrix} = Q \Lambda Q^T$$

where

$$Q = \begin{bmatrix} -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \text{ and } \Lambda = \begin{bmatrix} 10 & 0 \\ 0 & 8 \end{bmatrix}.$$

What is the singular value decomposition of A? Hint: If $A = UDV^T$ then $AV = UD$.