Quiz Problem 4

Assuming that we have training data $\{(x_n, y_n)\}_{n=1}^N$ where $x_n \in \mathbb{R}$ (i.e. $P = 1$) and y_n comes from one of two classes -1 or 1 encoded so that

$$y_n = \begin{cases} -1 & \text{if in class -1} \\ 1 & \text{if in class 1.} \end{cases}$$

Assume our training data has equal number of each class.

1. Consider training an LDA model on this data. Given a newly observed x, show that LDA predicts class 1 if and only if

$$\hat{\alpha_0} + \hat{\alpha} x > 0$$

   where

$$\hat{\alpha}_0 = -\frac{1}{2}(\hat{\mu}_1 + \hat{\mu}_{-1})(\hat{\mu}_1 - \hat{\mu}_{-1})$$

   and

$$\hat{\alpha} = \hat{\mu}_1 - \hat{\mu}_{-1}.$$

2. Fit a linear regression model of the $y_n s$ onto the $x_n s$ so that

$$y_n = \hat{\beta}_0 + \hat{\beta} x_n.$$

   Recall that for this type of simple linear regression

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}\bar{x}$$

   and

$$\hat{\beta} = \frac{\sum_{n=1}^N x_n(y_n - \bar{y}}{\sum_{n=1}^N (x_n - \bar{x})^2}.$$

   We can use this to build a classifier to classify points to class 1 if and only if

$$\hat{\beta}_0 + \hat{\beta} x > 0.$$

Show that this is equivalent to the LDA classifier. Hint: What is $\bar{y}$? What is $\bar{x}$ and $\sum_n x_n y_n$ in terms of $\hat{\mu}_1$ and $\hat{\mu}_{-1}$?