

Quiz Problem 6

Let y_n be a univariate response variable associated with some input $x_n \in \mathbb{R}^P$. Assume the probabilistic model where for fixed x_n we have $p(y_n | \beta)$ is the conditional distribution of y_n given β , so that

$$y_n | \beta \sim N(x_n^T \beta, \sigma^2)$$

and the y_n are independent. Assume that β has a Laplace distribution: $\beta_j \stackrel{iid}{\sim} \text{Laplace}(0, b)$ so that

$$p(\beta_j) = \frac{1}{2b} \exp\left(-\frac{|\beta_j|}{b}\right).$$

Consider finding $\hat{\beta}$ by maximizing $\log p(\beta | y_n)$ so that

$$\hat{\beta} = \arg \max_{\beta} \sum_{n=1}^N \log p(\beta | y_n).$$

Show that $\hat{\beta}$ is equivalent to the $\hat{\beta}^{(LASSO)}$ for some value of λ .