SP1

1. Check that
$$Q = \begin{bmatrix} 5/13 & 12/13 \\ -12/13 & 5/13 \end{bmatrix}$$
   is orthogonal

2. Find the eigenvalues of the matrix
$$A = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}.$$

3. Determine the eigenvectors of the matrix
$$A = \begin{pmatrix} 4 & 1 \\ 2 & 3 \end{pmatrix}.$$

4. Let
$$A = \begin{bmatrix} 9 & -2 \\ -2 & 6 \end{bmatrix}.$$

   (a) Is A symmetric?
   (b) Determine the eigen-values and eigen-vectors of A.
   (c) Find the spectral decomposition of A.
   (d) Find A^{-1} using the answer to the previous part.

5. Let $I_5 = diag(1, 1, 1, 1, 1)$ be the 5×5 identity matrix.

   (a) What is the spectral decomposition of I_5?
   (b) What is the singular value decomposition of I_5?

6. Let U be an orthogonal matrix. What is the singular value decomposition of U?

7. Let A be the following matrix
$$A = \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 2 & 1 \end{bmatrix}$$

   What is the SVD of A?

8. Find the singular values of the matrix
$$A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}.$$

9. If A is a symmetric matrix, show that the eigenvectors corresponding to distinct eigenvalues are orthogonal.

10. Let X be an $m \times n$ matrix. Prove that $X^T X$ is an $n \times n$ symmetric matrix.

11. For an $m \times n$ matrix X show that the non-zero eigenvalues of $X^T X$ and $X X^T$ are the same.