SP2

1 Q1 (ISLR 3.3)

2 Q2 (ISLR 3.6)

3 Q3 (ISLR 3.7)

4 Q4 (ISLR 3.10 a-c)

5 Q5 (ISLR 3.12)

6 Q6 (ISLR 3.13 a-f)

7 Q7

Let $X \in \mathbb{R}^{N \times P}$ be our design matrix and $Y = X\beta + \varepsilon$ where $\beta \in \mathbb{R}^P$. Let ε have a multivariate normal distribution so that $\varepsilon \sim N(0, \sigma^2 I)$ where $\sigma^2 > 0$ and I is the $N \times N$ identity matrix. Equivalently $Y \sim N(X\beta, \sigma^2 I)$. If our estimate of β is $\hat{\beta} = (X^T X)^{-1} X^T Y$ show that

$$\hat{\beta} \sim N(\beta, \sigma^2 (X^T X)^{-1}).$$

Hint: If $Z \sim N(\mu, \Sigma)$ where $\mu \in \mathbb{R}^N$ and $\Sigma \in \mathbb{R}^{N \times N}$ then if $B \in \mathbb{R}^{M \times N}$ we have $BZ \sim N(B\mu, B\Sigma B^T)$.

8 Q8

Argue that $RSS(\beta) = ||y - X\beta||^2 = y^T y - 2y^T X\beta + \beta^T X^T X\beta$.

9 Q9

Argue that $\frac{d}{d\beta}(y^T y) = 0$

10 Q10

Argue that $\frac{d}{d\beta}(y^T X\beta) = y^T X$

11 Q11

Argue that $\frac{d}{d\beta}(\beta^T X^T X\beta) = 2\beta^T X^T X$